# Learning from J apan: <br> A study of pedagogy that has led to high levels of mathematical thinking aptitude among J apan's <br> lower secondary school students 

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#### Abstract

Tests of secondary school students' mathematical aptitude have long played a central role in large-scale international comparative studies. While some researchers argue that this trend is due to the fact that mathematics possesses an agreed body of knowledge which can be easily compared between nations, a more plausible explanation comes from the undeniable significance that mathematical aptitude plays in commerce, industry, technology and science on which the wealth of all industrialised nations are built. Bearing that significance in mind, it is therefore no surprise that J apan's economic boom in the latter half of the twentieth century was preceded by a movement in its educational system typified by measurably high standards of mathematical aptitude. Extensive research into this phenomenon commonly concludes that it has been the result of constructivist pedagogy or unique socio-cultural circumstances; however, in this paper I present empirical evidence that challenges that assumption. I argue that much of what researchers in the West know about mathematics education in Japan has been coloured by linguistic barriers and the official line presented by J apan's Ministry of Education. Moreto the point, based on my research findings, I explain how factors fundamental toJ apan's educational success can be assimilated into almost any educational system.


The West has long shown an interest in J apan's educational system: as far back as 1900, just thirty years after J apan emerged from feudal rule, the Office of Special Reports of the Board of Education in London published a
paper on ‘Commercial Education in Japan'. However, prior to J apan's economic boom in the 1980s, the West's interest in J apan's educational system revolved around a belief that it was something that needed shaping and improving, and not something to learn from (Cummings et al., 1986).

Examination of J apan's educational achievements reveals that while students rarely excel on international tests of science or literacy-first or foreign language, they consistently excel on tests of mathematics. In fact, evidence suggests that Japan's lower secondary school students have outperformed their G8 counterparts in every major test of mathematical aptitude conducted since 1964. Unfortunately, however, according to H orio (1990), the involvement of Japan's Ministry of Education in Western research aimed at understanding this phenomenon has led to a body of research that is either misinformed or prejudiced. One such example of this is the unsubstantiated claims made by Her Majesty's Inspectorate (1991: 26) from the UK who reported that the West has little to learn from the $J$ apanese and their education system because of, among other reasons, cultural differences and their more homogeneous society.'

In order to understand the significance of factors that have led to Japan's lower secondary school students achieving a high level of mathematical aptitude, I first briefly detail the cognitive factors fundamental to the development mathematical aptitude. I explain how mathematical aptitude can be measured and consider how the wealth of a nation can be impacted by the standard of mathematical aptitude achieved by its lower secondary school students. I then turn my attention to mathematics education in J apan and show how various factors such as the intervention of J apan's Ministry of Education have led to a substantial amount of unclear and conflicting research findings. In the second part of this paper, I present independently collected survey data relating to the formal and non-formal mathematics education of lower secondary school students in 109 state, national and private schools in Hiroshima prefecture. I explain how my findings show that J apan's high instances of mathematical aptitude are not the product of sophisticated constructivist pedagogy as many believe but rather the outcome of a cognitive process that has occurred due to a combination of formal and non-formal mathematics instruction. Finally, I conclude this paper with a tentative proposal for how to create a similar learning environment within a formal educational system as a means
of raising mathematics standards.

## What is mathematical aptitude?

In spite of the fact that various schools of thought currently exist with regard to what pedagogical techniques best encourage the development of mathematical aptitude, common ground can still be found in the cognitive process—hereinafter referred to as mathematical thinking-that is believed to constitute that aptitude. Barnard (2002: 115), for example, defines mathematical thinking as:
...proceeding via operations involving a small number of 'items' at any onetime, an important feature is the phenomenon in which a section of mathematical structure may be mentally held as a single unit, processing an interiority that can be subsequently expanded without loss of detail and trigger connections with other parts of cognitive structure.

Tall (1994) similarly explains that mathematical thinking involves the construction of a mental object that can be manipulated in the mind by analogy with actions on objects experienced in the external world. More comprehensively, I would define mathematical thinking as the cognitive process that occurs when consolidated mathematical knowledge is applied indirectly to a problem by mapping the conceptual structure of one set of ideas into another (See Figure 1).

Figure 1: Mathematical Thinking (MT)


How is mathematical aptitude measured?

The International Association for the Evaluation of Educational Achievement (IEA) and the Organization for E conomic Cooperation and Development (OECD) currently dominate the world of large-scale international studies of secondary school students' mathematical ability. However, while each organization's studies bare a resemblance in that they centre around a test of mathematical ability, the source of the questions that appear in the respective tests differ: the IEA, on the one hand, draw questions from the curriculum of participating nations, whereas the OECD, in contrast, draw questions from real-world situations that do not appear in the curriculum of any of the participating nations.

Based on an understanding that a student who exhibits a high level of mathematical aptitude should be able to apply their mathematical knowledge to real-world situations using mathematical thinking, it can be argued that the OECD test is the more appropriate measure of that ability. However, that being said, this alone does not prove that the IEA test is not also equally as good at measuring mathematical aptitude. Owing to the fact that mathematical thinking is an essential element of mathematical modeling (See Figure 2) which is fundamental to most high-tech jobs on which the wealth of industrialised nations are built (Gainsburg, 2006), and considering that today's secondary school students will be part of tomorrow's workforce, it can be argued that the most accurate test of mathematical aptitude is one that closely correlates to the per capita wealth of a nation (See Table 1).

Figure 2: The Role of Mathematical Thinking (MT) in Mathematical Modeling


Table 1: Correlation of IEA and OECD mathematics studies against Gross National

| Year | Organisation | Mathematical <br> Study | Pearson correlation <br> measured against <br> GNI per capita |
| :---: | :---: | :---: | ---: |
| 1995 | IEA | TIMSS | $\mathbf{0 . 3 5 0}$ |
| 1999 | IEA | TIMSS-R | $\mathbf{0 . 5 7 5 ( * * )}$ |
| 2000 | OECD | PISA | $\mathbf{0 . 4 5 3 ( * )}$ |
| 2003 | IEA | FIMSS | $\mathbf{0 . 5 0 4 ( * * )}$ |
|  | OECD | PISA | $\mathbf{0 . 6 2 1 ( * * )}$ |

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

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\mathrm{n}=37
$$

TIMSS: Third International Mathematics and Science Study TIMSS-R: Third International Mathematics and Science Study-Repeat PISA: Program for International Student Assessment FIMSS: F ourth International Mathematics and Science Study

Source: Based on information from: the US Department of Education, National Center for Education Statistics. Available at: http://www.nctm.org; and World Development Indicators database, World Bank

As can be seen from Table 1 above, not only do the results of the OECD PISA 2003 test strongly correlate with participating nations' GNI per capita but so too do the IEA TIMSS-R 1999 and FIMSS 2003 tests; hence indicating the reliability of test data from both OECD and IEA studies as an accurate measure of lower secondary schools students' mathematical aptitude.

Although the Pearson coefficient does not allow for any direct conclusion about the causality of a correlation to be made, as Field (2005) explains it is possible to measure the variability of one variable that is explained by the other by squaring the coefficient and converting the resulting figure into a percentage (Figure 3). Consequently, as can be seen from Figure 3, the level of mathematical aptitude achieved by lower secondary school students is so closely tied to the wealth of a nation-accounting for anything up to 39 percent of GNI per capita-that
improving mathematics standards should be high on the list of priorities of any policymaker.

Figure 3: A measure of the impact lower secondary school students' mathematical aptitude has on GNI per capita


GNI per capita in 2003


Lower secondar'y schools students' mathematical aptitude as measured by the IEA FIMS 2003 test

GNI per capita in 2003
 mathematical aptitude as measured by the OECD PISA 2003 test

## Mathematical Aptitude in J apan

Bearing in mind that J apan, a country with few natural resources, emerged as the second largest economy in the latter half of the twentieth century on the back of its workforce, it comes as no surprise that the mathematical aptitude of its school students is measurably high. As previously noted, the mean average performance of J apan's lower secondary school students has been in excess of their G8 counterparts in every IEA and OECD test of mathematical aptitude conducted since 1964 (See Table 2 and Graph 1).

Table 2: Mean average performance of J apan's lower secondary school students on IEA
and OECD's tests of mathematical aptitude

1. IEA FIMS 1964
2. IEA SIMS 1981
3. IEA TIMSS 1995
4. IEA TIMSS-R 1999
5. IEA FIMSS 2003
6. OECD PISA 2000
7. OECD PISA 2003
$2^{\text {nd }}$ out of 12 countries
1st out of 20 countries
3 rd out of 41 countries
5 th out of 38 countries
5 th out of 46 countries

1 st out of 32 countries
6 6th out of 41 countries

FIMS: First International Mathematics Study
SIMS: Second International Mathematics Study

Source: Based on information from: the IEA available at http://www.iea.nl and MEXT (2005: 14)

Graph 1: G8 countries mean average performance of lower secondary school students in IEA and OE CD's comparative studies of mathematical aptitude conducted since 1995


Source: Based on information from: the US Department of Education, National Center for Education Statistics. Available at: http://www.nctm.org (See Appendix A for related data)

While there is no doubt about the high standard of mathematical aptitude achieved by J apan's lower secondary school students nor about the
impact those high standards have had on the J apanese economy, a number of issues of contention exist with regard to what factors have enabled that aptitude to develop; issues concerning the nature of pedagogy in secondary schools, the role of textbooks, the quality of educators, the motivational levels of students and the impact of J apan's non-formal educational system.

As previously mentioned, a number of inaccuracies in the findings of Western scholars about mathematics education in J apan have emerged due to the invol vement of the Ministry of Education (H orio, 1990). What is more, these inaccuracies are often compounded by one or more of the following factors:

I Scholars limited command of theJ apanese language
I Preconceived ideas about J apanese culture
I Limited understanding of mathematical thinking
I Preconceived ideas about what factors must be present in J apan in order for mathematical thinking to develop

For ease of future reference, I have collated summarised existing literature about mathematics education in J apan's lower secondary schools under the following headings: General/factual information (see Table 3); Non-conflicting positive findings (see Table 4); Non-conflicting negative findings (see Table 5); and Conflicting findings (see Table 6).

## Table 3: General/factual information about mathematics education in J apan

I J apan's lower secondary school students have outperformed their G8 counterparts in every IEA and OECD test of mathematical aptitude since 1964
I J apan's lower secondary school mathematics lessons follow a national curriculum
I J apan's lower secondary schools offer 105 ( 50 minute) mathematics lessons a year, which equates to 263 hours of mathematics instruction over the three years of lower secondary school education (As a point of comparison, approximately 330 hours of mathematics instruction is offered over three years of lower secondary school education in the UK)
I Mathematics is taught in mixed-ability classes
I The Japanese government invests less in education-as a percentage of GDP-than the OECD average (J apan 3.6 percent; OECD average 5.4 percent)
Table 4: Non-conflicting positive findings about education in J apan

I J apan's elementary school mathematics education is of the highest quality (J acobs and M orita, 2002; Whitburn, 2000; Stevenson and Lee, 1997; Rohlen, 1997)

Table 5: Non-conflicting negative findings about education in J apan
I Approximately 50 percent of J apan's lower secondary school students attend private cram schools (MEXT, 2005; Shimizu, 2001)

I J apan's ministry of education is keen to reform its current educational system (MEXT, 1998)

Table 6: Conflicting findings about education in J apan (Compare positive with negative)

| POSITIVE FINDINGS | NEGATIVE FINDINGS |
| :---: | :---: |
| I Mathematics lessons at lower secondary schools are taught using constructivist teaching methodology (Stigler and Hiebert, 1999; Jones, 1997; Whitman, 1991) | I Teachers at secondary schools in Japan use pedagogical techniques that encourage memorization and rote learning (Nemoto, 1999; White, 1987) <br> I Teachers at secondary schools teach to the test rather than trying to inspire students to think creatively (Schoppa, 1991) <br> I J apan's non-formal education compensates for the inflexibility of its formal educational system (Guo, 2005; Russell, 1997; Cummings, 1986; Kitamura, 1986) |
| I Textbooks are only used in approximately 2 percent of lower secondary school mathematics lessons ( ones, 1997) | I Teachers at secondary schools rely heavily on textbooks (Schmidt, 1993; Koyama, 1985) |
| I J apan has a wealth of highly qualified educators working at its lower secondary schools (Barrett, 2001) | I Japan's state secondary schools employ an excessively large number of part-time teachers-in some cases up to one third of the academic staff-many of whom are not qualified to work fulltime (Sakamoto, 2005) |
| POSITIVE FINDINGS | NEGATIVE FINDINGS |

I Entrance exams for upper secondary schools provide students with the extrinsic motivation required to attain a high level of mathematical aptitude (Guo, 2005; Rohlen, 1997; Shimahara, 1997)

I Japan's lower secondary school students have extremely low intrinsic and extrinsic motivation towards mathematics (OECD, 2004; Shimizu, 2001; Nemoto, 1999; Robitaille, 1993)

## METHOD

This study is the first part of a larger study that aims to provide valid and reliable informative about formal and non-formal mathematics education in J apan. In this paper, I investigate the following research questions:
(1) Are comparatively high instances of mathematical thinking among Japan's lower secondary school students the result of constructivist teaching methodology and limited use of textbooks in mathematics lessons?
(2) In what way has non-formal education in J apan aided the development of lower secondary school students' mathematical thinking?

## Participants and procedure

Data for this study was collected using a questionnaire survey comprising 16 closed answer questions (see Appendix B) about participants' experiences and opinions of lower secondary school formal and non-formal mathematics instruction. The participants came from 9 different academic departments at two universities in Hiroshima (see Appendix C) and between them graduated from a total of 143 different lower secondary schools across J apan: 111 of those schools-104 state, 5 private and 2 national—are located in Hiroshima prefecture; 12 schools-11 state and 1 national—are located in Yamaguchi prefecture; and 20 schools are located in 13 other prefectures.

The sample was chosen from university students for the following reasons:

I Data about pedagogy in a large number of lower secondary schools can be collected with a relatively small sample

I A survey can be carried out without the cooperation of local boards of education
I It is easy to guarantee university students the anonymity necessary for them to feel at ease to comment freely about mathematics teaching methodology in their lower secondary schools
I Due to the passing of time university students are more likely to objectively reflect on their lower secondary school experiences as a whole rather than subjectively comment on their current or most recent experience

## Measures

The questionnaire was divided into the following sections and subsections:

Section 1: About Yourself
Section 2: About Your Secondary Education
Subsection 1: Lower Secondary School: General Information
Subsection 2: Lower Secondary School: Mathematics
Section 3: About Non-F ormal Education

Each question on the questionnaire can be categorised as classification, behavioural or attitudinal. Classification questions-Q1-Q4, Q7-Q9 and Q13—split the sample into smaller subgroups for comparison. Behavioural questions-Q11-Q13—provide information about the mathematics teaching methodology and use of textbooks in participants' lower secondary schools; and about the participants' attendance of non-formal mathematics education. Attitudinal questions- Q5-Q6, Q10a/b Q14-Q16—provide information about participants' opinions concerning formal and non-formal mathematics education.

Due to the arbitrary nature of the values pertaining from the behavioural questions, I summarised the numeric data using frequencies. I then separated the sample into subgroups using the classification questions and tested for significance between two or more variables using Pearson's chi-square test. Finally, where a relationship was evident in a 95 percent confidence interval ( $p<0.05$ ), I used the odds ratio to measure the size of that effect. In the behavioural question relating to textbook use participants
were asked to categorise the frequency that textbooks were used in their lower secondary school mathematics lessons as Never, Rarely, Sometimes, Most Lessons or Every Lesson. In the question relating to teaching methodology participants were asked to tick the lesson plan-type A or $B$-that best resembled their typical lower secondary school mathematics lesson: type A lesson is based on a classical deductive mathematics lesson commonly practiced in the West, and type B lesson is based a constructivist mathematics lesson described by J acobs and M orita (2002: 167) as J apanese teachers' ideal script for a mathematics lesson; a third option, C was also presented for any participants who did not think that his or her mathematics lesson could be characterised by type A or type B.

After analysing participants' responses to the behavioural questions, I turned my attention to the attitudinal questions as a means of adding insight into the role that formal and non-formal mathematics education has played in raising standards of mathematical thinking. As with the behavioural questions, I tested for significance between two or more variables using Pearson's chi-square test and measured the size of the effect using the odds ratio.

## RESULTS

As can be seen from Graph 2, the key findings with regard to the use of textbooks in participants' lower secondary school mathematics lessons were as follows: only 3 percent of the entire sample reported that textbooks were rarely used in their mathematics lessons; a minimum of 80 percent or the participants in the four main subgroups-Hiroshima state schools, Hiroshima private schools, Yamaguchi private schools and schools in other prefecture-claimed that textbooks were used in most or every mathematics lesson; the most common (mode) response of participants in every statistically significant subcategory was that textbooks were used in every lesson; and the most typical (median: 50\% line) response in significant subcategories was that textbooks were used in most mathematics lessons. Finally, in a 95 percent confidence interval no statistically significant difference was found between the responses of participants in any of the subcategories.

Graph 2: A horizontal bar chart showing the frequency of textbook use in participants' lower secondary school mathematics


The key findings with regard to mathematics teaching methodol ogy were as follows (see Table 7): every participant that responded indicated that their typical lower secondary school mathematics lesson was either deductive or constructivist; only 6 percent of the entire sample indicated that their lower secondary school mathematics lessons were constructivist whereas 93 percent indicated that their lessons were deductive in nature; a maximum of 9 percent of participants in any statistically significant subcategories indicated that their mathematics lessons were constructivist; and no statistically significance difference was found between the responses of the participants in any of the subcategories.

Based on the above findings, it follows that as there is a 3 in 50 chance of a lesson picked at random from the sample being constructivist and a 3 in 100 chance of a lesson being one in which a textbook is rarely used, there is only 9 in 5000 chance that a lesson picked at random is constructivist in which a textbook is rarely used. This figure is supported by the fact that only one of the 264 participants in the sample indicated that his or her mathematics lessons satisfied both of those criteria.

Table 7: A contingency table showing how the participants categorised the teaching methodology in their lower secondary school mathematics lessons

|  |  | A: Lower Secondary School by region/type |  |  |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  | Deductive <br> \% of A Total | $\begin{aligned} & 187 \\ & 93 \% \end{aligned}$ | $\begin{gathered} \hline 20 \\ 95 \% \end{gathered}$ | $\begin{gathered} \hline 2 \\ 67 \% \end{gathered}$ | $\begin{gathered} 11 \\ 92 \% \end{gathered}$ | $\begin{gathered} 1 \\ 100 \% \end{gathered}$ | $\begin{gathered} 19 \\ 95 \% \end{gathered}$ | $\begin{gathered} 5 \\ 100 \% \end{gathered}$ | $\begin{aligned} & \hline 245 \\ & 93 \% \end{aligned}$ |
|  | Constructivist \% of A Total | $\begin{aligned} & 13 \\ & 6 \% \end{aligned}$ | $\begin{gathered} 1 \\ 5 \% \end{gathered}$ | $\begin{gathered} 1 \\ 33 \% \end{gathered}$ | $\begin{gathered} 1 \\ 8 \% \end{gathered}$ |  | $\begin{gathered} 1 \\ 5 \% \end{gathered}$ |  | $\begin{aligned} & 17 \\ & 6 \% \end{aligned}$ |
|  | No Entry \% of A Total | $\begin{gathered} 2 \\ 1 \% \end{gathered}$ |  |  |  |  |  |  | $\begin{gathered} \mathbf{2} \\ \mathbf{1 \%} \end{gathered}$ |
|  | Total Count \% of Total | $\begin{aligned} & 202 \\ & 77 \% \end{aligned}$ | $\begin{gathered} 21 \\ 8 \% \end{gathered}$ | $\begin{gathered} \mathbf{3} \\ 1 \% \end{gathered}$ | $\begin{aligned} & 12 \\ & 5 \% \end{aligned}$ | $\begin{gathered} 1 \\ <\mathbf{1} \% \end{gathered}$ | $\begin{gathered} 20 \\ 8 \% \end{gathered}$ | $\begin{gathered} \hline 5 \\ 2 \% \end{gathered}$ | $\begin{gathered} 264 \\ 100 \% \end{gathered}$ |

Other notable findings include:
I Participants who were taught mathematics using constructivist methodology were no more statistically likely to like or dislike their lesson than participants who were taught using deductive methodology
I 67 percent of the sample studied mathematics at cram school when they were lower secondary school students
I Participants who studied mathematics at cram school were 2.35 times more likely to think that their school mathematics lessons were easier than those who did not
I 62 percent of the participants who attended cram school think that their cram school mathematics lessons were more interesting than their lower secondary school mathematics lessons
I 84 percent of the participants that attended cram school think that their cram school mathematics lessons were more useful in hel ping them to prepare for the upper secondary school entrance exams than
their lower secondary school mathematics lessons
I 81 percent of the participant who attended cram school think that their cram school lessons were easier to understand than their lower secondary school mathematics lessons
I 32 percent of the students who went on to study a major at university in which mathematical aptitude is important were able to do so without studying mathematics at cram school
I Participants who studied mathematics at cram school were no more likely to go on to study a major at university in which mathematical aptitude is important than those who did not

## DISCUSSION

Based on the data collected in this study, the simple answer to the question, 'Are comparatively high instances of mathematical thinking among J apan's lower secondary school students the result of constructivist teaching methodology and limited use of textbooks in mathematics lessons?' is no; at least not in Hiroshima prefecture for which this study includes information about teaching practices in 70 percent of the state schools. What is more, with 93 percent of the sample indicating that their mathematics lessons were deductive in nature (see Table 7) and 83 percent (see Graph 2) that textbooks were used in most or every mathematics lesson, it can be argued that the lower secondary school mathematics lessons experienced by the participants in this study were, for all intents and purposes, the same as the mathematics lessons experienced by their counterparts in the West.

After dismissing the idea that constructivist pedagogy has been fundamental to J apan's high standards of mathematical thinking, attention must turn to understanding the importance of non-formal mathematics instruction at private cram schools which approximately 50 percent of lower secondary school students attend (MEXT, 2005). If the findings of this questionnaire survey are taken at face value, it would appear that mathematics instruction in J apan's private cram schools is more interesting, easier to understand, and prepares students for high school entrance exams better than the mathematics instruction at lower secondary schools. However, this kind of statement overlooks the fact that mathematics instruction at private cram schools is not a primary source of educational input. As a secondary source of educational input cram schools benefit from
the fact that most students arrive with prior mathematical knowledge that aids in making the learning experience more interesting and easier to understand. Furthermore, while there is no doubt that some cram schools employ dynamic teachers who teach using sophisticated pedagogical practices, the vast majority of teachers in cram schools are not so called 'star' teachers; in fact, as Roesgaard (2006) points out more than 80 percent of teachers in cram schools have never had any kind of formal teaching training.

Analysis of the importance of mathematics instruction at cram schools may look like a daunting task owing to its extensive variety in terms of class size, use of streaming, use of technology and the experience or qualifications of the teachers; however, it is that variety that makes it so much easier to identify the function it serves in raising students mathematical thinking. After eliminating all inconsistencies, only three significant factors that influence the learning experience of students attending private crams schools remain:

1. Change in environment
2. Change in classroom dynamic
3. Change in perspective

Consequently, it can be argued that the act of transposing mathematical knowl edge from the primary learning environment to the secondary learning environment where that knowledge must then be used to interact with a similar mathematical problem from a potentially different perspective in a classroom with a different dynamic requires the mapping of conceptual structure of one set of ideas into another and vice versa (see Figure 4); hence aiding in the consolidation of mathematical knowledge through a basic form of mathematical thinking that would not be possible if students were to spend more time study the same mathematical problems in the primary learning environment.

Finally, it should be noted that while some students are naturally apt at mapping the conceptual structure of one set of ideas into another, the vast majority are not. Therefore, if any nation hopes to benefit from the obvious advantages of a workforce with high levels of mathematical thinking without developing a large non-formal educational sector, it needs to consider how to
incorporate a change of environment, perspective and classroom dynamic into its current educational system.

Figure 4: A map of mathematical thinking (MT) in J apan


## IMPLICATIONS, LIMITATIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

The results of this study suggest that comparatively high instances of mathematical thinking among J apan's lower secondary school students are not the result of constructivist teaching methodology and limited use of textbooks in mathematics lessons as some researchers believe, but rather the result of a combination of formal and non-formal mathematics instruction. I use the results to explain how a combination of studying mathematics in two different environments, with two different classroom dynamics and from two potentially different perspectives requires students to map the conceptual structure of one set of ideas into another, which is fundamental to the development of mathematical thinking. The most significant implication of this paper results from the identification of underlying factors that have enabled mathematical thinking to develop in J apan is that it provides other
nations a means of creating a comparable learning environment within their own formal educational system.

Due to the limitations of time and space, I have only presented one aspect of a much larger study about J apan's mathematics education in this paper. In a follow up to this study, I confirm the findings presented in this paper with questionnaires, interviews and classroom observations invol ving lower secondary school students, teachers and members of the local boards of education; I compare and contrast the contents of the mathematics curriculum taught in J apan's lower secondary schools with other G8 nations; I discuss the significance of the high number of J apan's lower secondary school students staying on in fulltime education until they are 18 years old; and I look in detail at how J apan has benefited from having a system of examinations at the entrance to almost every stage of education.

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## Appendix A

|  | IEA <br> TIMSS 1995 <br> $(41)$ | IEA <br> TIMSS-R 1999 <br> $(38)$ | IEA <br> FIMSS 2003 <br> $(46)$ | OECD <br> PISA 2000 <br> $(32)$ | OECD <br> PISA 2003 <br> $(41)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Canada | $521(11)$ | $531(10)$ |  | $533(5)$ | $532(7)$ |
| England | $498(14)$ | $496(20)$ |  | $529(7)$ |  |
| France |  |  |  | $517(10)$ | $511(17)$ |
| Germany |  |  |  | $490(20)$ | $503(20)$ |
| Italy | $491(16)$ | $479(22)$ | $484(22)$ | $457(26)$ | $466(29)$ |
| J apan | $581(3)$ | $579(5)$ | $570(5)$ | $557(1)$ | $534(6)$ |
| Russian Federation | $524(10)$ | $526(12)$ | $508(10)$ | $478(22)$ | $468(28)$ |
| United States | $492(15)$ | $502(19)$ | $504(15)$ | $493(19)$ | $483(27)$ |

Source: Based on information from: the US Department of Education, National Center for Education Statistics. Available at: http://www.nctm.org

## Appendix B



Q5. Do you think that mathematical aptitude is important for your major?


Q6. Do you think that mathematical aptitude is important for your future career?

$\square$ No

## About Your Lower Secondary School E ducation

Q7. What lower secondary school did you goto? $\qquad$

Q8. What prefecture is it in?
$\square$ Hiroshima $\square$ Other (please specify) $\qquad$

Q9. What type of school is it?
$\square$ State $\quad \square$ National $\quad \square$ Private

Q10.a. Did you like your mathematics lessons in lower secondary school?
 Yes


Q10.b. Why? (Tick as many boxes as is appropriate)
 The lesson was interesting The lesson was easy You liked the teacher
 The lesson was boring The lesson was difficult You disliked the teacher Other (please specify) $\qquad$ You disiked theteacher

Q11. How often was a textbook used in the lesson?


Never

Rarely

Sometimes

M ost Lessons

Every Lesson

Q12. Which lesson plan best describes your lower secondary school mathematics lesson?

$\qquad$
$\qquad$
$\qquad$

| TypeA Mathematics Lesson | Type B Mathematics Lesson |
| :--- | :--- |
| The teacher tells the class what <br> mathematics they are going to study. | The teacher poses a new mathematical <br> problem on the board. |
| The teacher explains how to solve that type <br> of mathematics problem using an example <br> from the textbook. | In groups: Students discuss the <br> problem until they agree on a solution. |
| The teacher writes a similar mathematics <br> question on the board which students work <br> on individually. | Students present their sol ution to the <br> problem in front of the class. |
| A small number of students are called to the <br> front to write their answer on the board <br> which is then checked by the teacher and <br> used to explain how to answer the question. | The teacher guides the students to the <br> best solution. |
| Students work individually answering <br> similar questions in the textbook. | Students use what they have learned <br> to solve similar problems individually. |

## About Non-Formal Education

Q13. Did you study mathematics at a private cram school when you were in lower secondary school?


Yes $\square$ No
If you answered yes to the above question, in which year of study at lower secondary school did you also attend private cram school? (Tick as many boxes as is appropriate)
$\square$ First $\square$ Second $\square$ Third

## If you answered yes to Q13 please answer Q14~Q16

Q14. Which mathematics lesson did you enjoy more? (Tick one only)
$\square$ Cram School $\square$ Lower Secondary School

Q15. Which mathematics lesson was more helpful in preparing for the upper secondary school entrance exam? (Tick one only)


Q16. Which mathematics lesson was easier to understand? (Tick one box)
$\square$ Cram School $\square$ Lower Secondary School

## Appendix C

|  | First Year |  | Second Year |  | Third Year |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Female | Male | Female | Male | Female | Male | Total |
| HCU Intelligent Systems |  |  | 11 | 13 |  |  | $\mathbf{2 4}$ |
| HCU Info Machines and Interfaces |  |  | 1 | 23 |  |  | $\mathbf{2 4}$ |
| YWU E nglish | 38 |  | 13 |  | 8 |  | $\mathbf{5 9}$ |
| YWU J apanese Literature | 16 |  | 1 |  |  |  | $\mathbf{1 7}$ |
| YWU Psychology | 22 |  | 8 |  |  |  | $\mathbf{3 0}$ |
| YWU Primary E ducation | 25 |  |  |  |  |  | $\mathbf{2 5}$ |
| YWU Business |  |  | 13 |  | 6 |  | $\mathbf{1 9}$ |
| YWU Design | 21 |  | 6 |  |  |  | $\mathbf{2 7}$ |
| YWU Nutrition | 28 |  | 11 |  |  |  | $\mathbf{3 9}$ |
| Total | 150 | 0 | 64 | 36 | 14 | 0 | $\mathbf{2 6 4}$ |

HCU = Hiroshima City University
YWU =Yasuda Women's University

